Girraween High School

2025 (Term 4)

Year 12 HSC Task 1

Mathematics Extension 1

General Instructions

- Reading time: 5 minutes
- Working time: 60 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Answer multiple-choice questions on the multiple-choice answer sheet provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 43

Section I

Multiple Choice (5 Marks)

- Attempt Questions 1 -5
- Allow about 10 minutes for this section

Section II

Extended Response (38 Marks)

- Attempt Questions 6 - 11
- Allow about 50 minutes for this section

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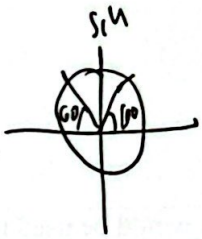
Section I: Multiple Choice

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 5.

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1. What is the value of $\sin 2x$ given that $\sin x = \frac{2\sqrt{3}}{4}$ and x is obtuse?



$x = 60$

(A) $\frac{\sqrt{3}}{4}$

(B) $\frac{\sqrt{3}}{2}$

$x = 180 - 60$
$= 120$

(C) $-\frac{\sqrt{3}}{4}$

(D) $-\frac{\sqrt{3}}{2}$

$\sin 2x120$
$= \sin 240$

2. If $t = \tan\frac{x}{2}$ which of the following shows the equation being solved if:

$$\left(\frac{6t}{1+t^2}\right)+\left(\frac{4t^2-4}{1+t^2}\right)=-2$$

(A) $4\sin x - 3\cos x = -2$

(B) $3\sin x - 4\cos x = -2$

(C) $3\tan x + 4\cos x = -2$

(D) $3\sin x + 4\cos x = -2$

$3\sin x - 4\cos x$

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3. Given $\cos 2\alpha = -2\sin^2 \alpha - \sin 2\alpha$, find the value of $\tan \alpha$.

(A) -1

(B) 1

(C) $\frac{\pi}{4}$

(D) $\frac{3\pi}{4}$

4. Which expression would be used to solve the equation $4 \cos x = \operatorname{cosec} x$?

(A) $\sin 2x = \frac{1}{4}$

(B) $\sin 2x = \frac{1}{2}$

(C) $\cos 2x = \frac{1}{4}$

(D) $\cos 2x = -\frac{1}{2}$

Section 1 continues over page

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5. Mathematical induction is used to prove:

$$\frac{2}{1\times 2}+\frac{2}{2\times 3}+\frac{2}{3\times 4}+\ldots+\frac{2}{n\times(n+1)}=\frac{2n}{n+1} \text{ for all positive integers } n.$$

Which of the following is the correct expression for part of the induction proof?

(A) $\text{LHS}=\frac{2}{1\times 2}+\frac{2}{2\times 3}+\frac{2}{3\times 4}+\ldots+\frac{2}{k\times(k+1)}$

$=\frac{2k}{k+1}+\frac{2}{(k+1)}$

(B) $\text{LHS}=\frac{2}{1\times 2}+\frac{2}{2\times 3}+\frac{2}{3\times 4}+\ldots+\frac{2}{(k+1)}+\frac{2}{(k+2)}$

$=\frac{2k}{k+1}+\frac{2}{(k+2)}$

(C) $\text{LHS}=\frac{2}{1\times 2}+\frac{2}{2\times 3}+\frac{2}{3\times 4}+\ldots+\frac{2}{k\times(k+1)}+\frac{2}{(k+1)(k+2)}$

$=\frac{2k}{k+1}+\frac{2}{(k+1)(k+2)}$

(D) $\text{LHS}=\frac{2}{1\times 2}+\frac{2}{2\times 3}+\frac{2}{3\times 4}+\ldots+\frac{2}{k\times(k+1)}+\frac{2}{(k+1)(k+3)}$

$=\frac{2k}{k+1}+\frac{2}{(k+1)(k+3)}$

End of Section I

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Section II

38 Marks
Attempt Questions 6 - 11
Allow about 50 minutes for this section.

Answer each question on a new page in the writing booklet. Extra writing booklets are available. For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (7 Marks) Start a new page in the writing booklet. **Marks**

a) Use mathematical induction to prove that the following is true for all integers $n \geq 1$. **4**

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$$

b) Solve $\tan 2x + \tan x = 0$ in the domain $0 \leq x \leq \pi$. **3**

Question 7 (5 Marks) Start a new page in the writing booklet.

a) i) Express $2\cos x + 2\sqrt{3}\sin x$ in the form $R\cos(x - \alpha)$ where $R > 0$ and α is acute. **3**

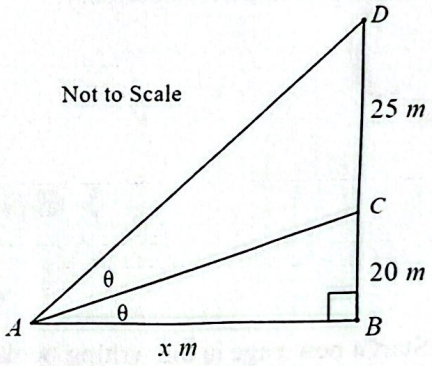
ii) Hence, solve $2\cos x + 2\sqrt{3}\sin x = 2$ in the domain $0 \leq x \leq 2\pi$. **2**

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Question 8 (7 marks) Start a new page in the writing booklet.

a) Use mathematical induction to prove that $7^n - 3^n$ is divisible by 4 for all integers $n \geq 1$. **4**

b) Daniel is standing at a bus stop, at A, and notices a sign on the building opposite at C, which is 20 m above the ground. When he doubles the angle of elevation to the building opposite, he notices a person looking out of the window 45 m above the ground level, at D. B is the base of the building.



i) Show that $\frac{45}{x} = \frac{40x}{x^2 - 400}$. **2**

ii) How far is the building from where Daniel is standing? **1**

Question 9 (6 marks) Start a new page in the writing booklet.

a) If $t = \tan\frac{\theta}{2}$, solve $2\cos\theta + \sin\theta = -1$ in the domain $0 \leq \theta \leq 2\pi$. **3**

b) If $x = \sin A$ and $x = \cos B$, solve $\sin(A - B) = 3x + 1$. **3**

SOLA

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Question 10 (7 marks) Start a new page in the writing booklet.

a) Prove by mathematical induction that $\sum_{r=1}^{n} \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$ for all integers $n \geq 1$. **4**

b) Solve $\sin\left(x + \frac{\pi}{3}\right) = \cos x$ for $0 \leq x \leq 2\pi$. **3**

Answer to 2 decimal places where necessary.

Question 11 (6 marks) Start a new page in the writing booklet.

a) i) Show that $\frac{1}{4}(\cos 9x + \cos 7x + \cos 5x + \cos 3x) = \cos 6x \cos 2x \cos x$. **3**

ii) Hence, find all the solutions of: **3**

$$\frac{1}{4}(\cos 9x + \cos 7x + \cos 5x + \cos 3x) = 0 \quad \text{for } 0 \leq x \leq \pi.$$

End of Assessment Task

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Solutions & Feedback

Mathematics Assessment Solutions
Girraween High School

page

☒ Extension 1
☐ Extension 2

Year 2026
Task Number 1

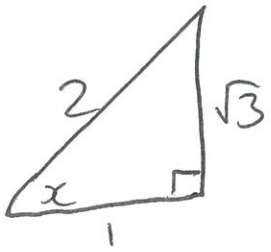
Suggested solutions	Marker's Feedback

D
B
A
B
C

Multiple Choice

1, $\sin x = \frac{2\sqrt{3}}{4}$

$\sin x = \frac{\sqrt{3}}{2}$



$\sin 2x = 2\sin x \cos x$

$= 2 \times \frac{\sqrt{3}}{2} \times -\frac{1}{2}$

$= -\frac{\sqrt{3}}{2}$

(D)

2, $\frac{6t}{1+t^2} + \frac{4t^2-4}{1+t^2} = -2$

$3\left(\frac{2t}{1+t^2}\right) - 4\left(\frac{1-t^2}{1+t^2}\right) = -2$

$3\sin x - 4\cos x = -2$

(B)

Suggested solutions	Marker's Feedback

3, $\cos 2\alpha = -2\sin^2\alpha - 2\sin 2\alpha$

$\cos^2\alpha - \sin^2\alpha = -2\sin^2\alpha - 2\sin\alpha\cos\alpha$

$\cos^2\alpha + 2\sin\alpha\cos\alpha + \sin^2\alpha = 0$

$(\cos\alpha + \sin\alpha)^2 = 0$

$\cos\alpha + \sin\alpha = 0$

$\sin\alpha = -\cos\alpha$

$\tan\alpha = -1$

(A)

4, $4\cos x = \operatorname{cosec} x$

$4\cos x = \frac{1}{\sin x}$

$4\cos x \sin x = 1$

$2\sin x\cos x = \frac{1}{2}$

$\sin 2x = \frac{1}{2}$

(B)

5, (C)

Suggested solutions	Marker's Feedback

Q6a) $1^3+2^3+\ldots+n^3=\frac{1}{4}n^2(n+1)^2$

1, Prove true for $n=1$

$LHS=n^3$ $RHS=\frac{1}{4}n^2(n+1)^2$

$=1^3$ $=\frac{1}{4}\times 1^2(1+1)^2$

$=1$ $=\frac{1}{4}\times 2^2$

$=1$

$\therefore$ true for $n=1$ $=LHS$

(1) - Prove true $n=1$

(1) - assumption

(1) - using assumption correctly

(1) - Proving for $n=k+1$

2, Assume true for $n=K$

$1^3+2^3+3^3+\ldots+K^3=\frac{1}{4}K^2(K+1)^2$ (1)

3, Prove true for $n=K+1$

RTP $1^3+2^3+\ldots+K^3+(K+1)^3=\frac{1}{4}(K+1)^2(K+2)^2$

$LHS=1^3+2^3+\ldots+K^3+(K+1)^3$

$=\frac{1}{4}K^2(K+1)^2+(K+1)^3$ using assumption (1)

$=\frac{1}{4}(K+1)^2(K^2+4(K+1))$

$=\frac{1}{4}(K+1)^2(K^2+4K+4)$

$=\frac{1}{4}(K+1)^2(K+2)^2$

$=RHS$ ✓ $\therefore$ true for $n=K+1$

$\therefore$ By mathematical induction the statement is true

done well

Suggested solutions	Marker's Feedback

6b) $\tan 2x + \tan x = 0 \quad 0 \le x \le \pi$

$$\frac{2\tan x}{1-\tan^2 x} + \tan x = 0$$

$$2\tan x + \tan x(1-\tan^2 x) = 0$$

$$2\tan x + \tan x - \tan^3 x = 0$$

$$\tan^3 x - 3\tan x = 0$$

$$\tan x(\tan^2 x - 3) = 0$$

$\tan x = 0$ $\quad \tan^2 x = 3$

$x = 0, \pi$ $\quad \tan x = \pm\sqrt{3}$

related angle $x = \frac{\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

Common mistakes

* Solving only $\tan^2 x - 3 = 0$
* Solving $\tan x = \pm 3$

(1) $\tan x(\tan^2 x - 3) = 0$

(1) - solving $\tan x = 0$ correctly

(1) - solving $\tan^2 x = 3$ correctly

Some wrote the solutions

$0, \pi, \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

They were not penalised.

Suggested solutions	Marker's Feedback

7a)(i) $2\cos x + 2\sqrt{3}\sin x$

$R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$

① $R\cos\alpha = 2$ $\quad R\sin\alpha = 2\sqrt{3}$ ②

(1) – equating coefficients of $\cos x$ & $\sin x$

$R^2 = 2^2 + (2\sqrt{3})^2$

$R^2 = 4 + 4\times 3$

$R^2 = 16$

$R = \pm 4 \quad \therefore R = 4$ as $R > 0$

(1) – correct value for R

$\therefore$ ① $4\cos\alpha = 2$ $\quad$ ② $4\sin\alpha = 2\sqrt{3}$

$\cos\alpha = \frac{1}{2}$ $\quad \sin\alpha = \frac{\sqrt{3}}{2}$

Both positive, $\therefore$ 1st quadrant

$\therefore \alpha = \cos^{-1}\frac{1}{2}$ or $\alpha = \sin^{-1}\frac{\sqrt{3}}{2}$

$\alpha = \frac{\pi}{3}$ $\quad \alpha = \frac{\pi}{3}$

(1) – correct angle
– verifying quadrants
– final answer

$\therefore 2\cos x + 2\sqrt{3}\sin x = 4\cos\left(x - \frac{\pi}{3}\right)$

*Very well done

7a)

Suggested solutions

(ii) $4\cos\left(x-\frac{\pi}{3}\right)=2$ $\quad 0\le x\le 2\pi$

$\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{2}$ $\quad \Downarrow$

$-\frac{\pi}{3}\le x\le\frac{5\pi}{3}$

Change domain

Related angle

$x-\frac{\pi}{3}=\frac{\pi}{3}$

+	
+	

$\therefore x-\frac{\pi}{3}=-\frac{\pi}{3},\frac{\pi}{3},\frac{5\pi}{3}$

$\boxed{x=0,\frac{2\pi}{3},2\pi}$

Marker's Feedback

(1) correct equation & changing domain

(1) - correct solutions

(-1) per error

* Some sts did not change the domain so they missed $x=0$ as a solution

Suggested solutions	Marker's Feedback

8a) $7^n - 3^n$ divisible by 4, $n \geq 1$

Prove true for $n=1$

$7^1 - 3^1 = 4$

which is divisible by 4 ← (1) Prove true for $n=1$

$\therefore$ true for $n=1$

Assume true for $n=k$

$\therefore 7^k - 3^k = 4M$ (M is an integer)

$7^k = 4M + 3^k$ (1) ← (1) Assumption correct

Prove true for $n=k+1$

RTP: $7^{k+1} - 3^{k+1} = 4P$ (P is an integer)

Very well done

LHS $= 7^{k+1} - 3^{k+1}$

$= 7^k_{\times} 7 - 3^k_{\times} 3$

$= 7(4M + 3^k) - 3^k_{\times} 3$ (using assumption (1)) ← (1) using assumption

$= 28M + 7_{\times} 3^k - 3_{\times} 3^k$

$= 28M + 4_{\times} 3^k$

$= 4(7M + 3^k)$ ← (1) proving true

$= 4P \Leftarrow$ RHS

$\therefore$ divisible by 4 as M & P & K are integers

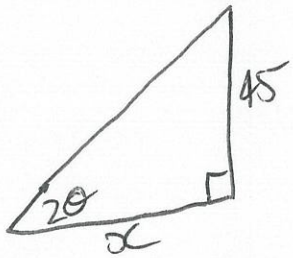
$\therefore$ true for $n=k+1$

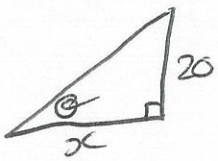
$\therefore$ By Mathematical induction the statement is true.

Suggested solutions	Marker's Feedback

8b)

i)





$\tan 2\theta = \frac{45}{x}$ $\tan\theta = \frac{20}{x}$

Generally well done

$\therefore \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(1) correct, sub into tan 2θ

$\frac{45}{x} = \frac{2\times\left(\frac{20}{x}\right)}{1-\left(\frac{20}{x}\right)^2}$

$\frac{45}{x} = \frac{40}{x} \times \frac{x^2}{x^2-400}$

$\frac{45}{x} = \frac{40x}{x^2-400}$ as required ← (1) correctly showing this

(ii) $45(x^2-400) = 40x^2$

$45x^2 - 18000 = 40x^2$

$5x^2 = 18000$

$x^2 = 3600$

$x = \pm 60$

← (1) correct answer

well done

$\therefore x = 60\text{m}$ as $x > 0$

Suggested solutions	Marker's Feedback

$0 \le \theta \le 2\pi$

9a) $2\cos\theta + \sin\theta = -1$

$2\left(\frac{1-t^2}{1+t^2}\right) + \frac{2t}{1+t^2} = -1$ ← (1) - correct substitution

* This part generally well done.

$2 - 2t^2 + 2t = -(1+t^2)$

$2 - 2t^2 + 2t = -1 - t^2$

$0 = t^2 - 2t - 3$

$0 = (t+1)(t-3)$

$\therefore t = -1$ and 3 ← (1) correct values of t

* A few people didn't restrict the domain of $\frac{\theta}{2}$ to $0 \le \frac{\theta}{2} \le \pi$ which resulted in them getting extra answers.

$0 \le \frac{\theta}{2} \le \pi$

$\therefore \tan\frac{\theta}{2} = -1$

$\tan\frac{\theta}{2} = 3$

related angle $= \frac{\pi}{4}$

related angle $= 1.25$ rad

$\therefore \frac{\theta}{2} = \frac{3\pi}{4}$ &

$\frac{\theta}{2} = 1.25^c$

$\theta = \frac{6\pi}{4}$

$\therefore \theta = 2.50^c$

$\theta = \frac{3\pi}{2}$

In degrees

$\frac{\theta}{2} = 71.6°$

$\theta = 143.2°$, ← (1) correct angles

Not penalised but mentioned.

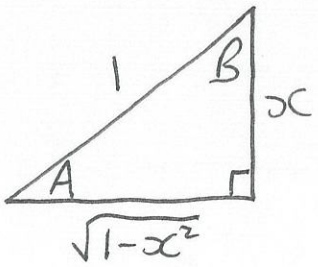
Test: $\theta = \pi$: $2\cos\pi + \sin\pi = -2 \neq -1$.

π is not a solution.

$\theta = \frac{3\pi}{2}, 2.5$

Suggested solutions	Marker's Feedback

9b) $x = \sin A \quad x = \cos B$



Students had a LOT of trouble seeing what to do.

Quite a few students could see this part

$\sin(A-B) = 3x+1$

$\sin A\cos B - \sin B\cos A = 3x+1$ ✓

$x \times x - \sqrt{1-x^2} \times \sqrt{1-x^2} = 3x+1$ ✓

(1) correct use of sin compound angles

$x^2 - (1-x^2) = 3x+1$

$x^2 - 1 + x^2 = 3x+1$

$2x^2 - 3x - 2 = 0$

$(x-2)(2x+1) = 0$

$\therefore x = 2$ and $-\frac{1}{2}$

But very few realised that if $x = \sin A$ $\cos A = \sqrt{1-x^2}$ & if $x = \cos B$, $\sin B = \sqrt{1-x^2}$.

(1) - correct substitution

Some students MISQUOTED THIS VERY BADLY.

Now $-1 \leq x \leq 1$ $\therefore$ $\boxed{x = -\frac{1}{2}}$

(1) correct ans ✓

Very few students realised that $x \neq 2$ as $-1 \leq \sin A \leq 1$.

A very common (and VERY bad) mistake was for students to get to $x^2 - \sin B\cos A = 3x+1$

$\therefore x^2 - 3x - 1 = \sin A\cos B$

then solving $x^2 - 3x - 1 = 0$?

This was a VERY bad mistake. as $\sin A\cos B \neq 0$.

As I wrote, it's the NULL FACTOR LAW NOT the $\sin B\cos A$ FACTOR LAW!

Suggested solutions	Marker's Feedback

10a) $\sum_{r=1}^{n} \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$, $n \geq 1$

Prove true for $n=1$

LHS $= \frac{1 \times 2^1}{(1+2)!}$ RHS $= 1 - \frac{2^{1+1}}{(1+2)!}$

$= \frac{2}{3!}$ $= 1 - \frac{4}{6}$

$= \frac{1}{3}$ $= \frac{1}{3}$

$= LHS$

(1) - Prove true for $n=1$

$\therefore$ true for $n=1$

Assume true for $n=K$

i.e. $\frac{1}{3} + \ldots + \frac{K \times 2^K}{(K+2)!} = 1 - \frac{2^{K+1}}{(K+2)!}$ (1)

(1) correct assumption

Prove true for $n=K+1$

R.T.P $\frac{1}{3} + \ldots + \frac{K \cdot 2^K}{(K+2)!} + \frac{(K+1) \times 2^{K+1}}{(K+1+2)!} = 1 - \frac{2^{K+2}}{(K+3)!}$

LHS $= \frac{1}{3} + \ldots + \frac{K \cdot 2^K}{(K+2)!} + \frac{(K+1) \times 2^{K+1}}{(K+3)!}$

$= 1 - \frac{2^{K+1}}{(K+2)!} + \frac{(K+1) \times 2^{K+1}}{(K+3)!}$ using assumption (1)

(1) using assumption

$= 1 - \frac{(K+3)2^{K+1} + (K+1)2^{K+1}}{(K+3)!}$

Many students made mistakes at this step because of the negative sign before 2^{k+1}

$= 1 - \frac{2^{K+1}(K+3-K-1)}{(K+3)!}$

$= 1 - \frac{2^{K+1} \times 2}{(K+3)!}$

(1) - proving true for $n=k+1$

$\therefore$ true for $n=k+1$

$= 1 - \frac{2^{K+2}}{(K+3)!}$

$= RHS$

$\therefore$ By mathematical induction The statement is true.

Suggested solutions	Marker's Feedback

10b) $\sin\left(x+\frac{\pi}{3}\right)=\cos x,\ 0\le x\le 2\pi$

$\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}=\cos x$

$\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\cos x$ ← (1) Compound sin with exact values substituted

$\frac{1}{2}\sin x=\left(1-\frac{\sqrt{3}}{2}\right)\cos x$

$\frac{\sin x}{\cos x}=2\left(1-\frac{\sqrt{3}}{2}\right)$

$\tan x=2-\sqrt{3}$ ← (1) - $\tan x$

+
+

as $\tan x>0$

related angle $x=0.261799$ rad

$x\approx 0.26$ rad

$\therefore\ x=0.26^\circ,\ 3.40^\circ$ ← (1) - correct answers

$(\pi+0.26)$

Some students have trouble solving the equation. Some didn't give the answers correct to 2dp but didn't penalise for that.

Suggested solutions	Marker's Feedback

11a) Product to Sums

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$\frac{1}{4}(\cos 9x + \cos 7x + \cos 5x + \cos 3x) = \cos 6x \cos 2x \cos x$

$LHS = \frac{1}{4}(\cos(8x+x) + \cos(8x-x) + \cos(4x+x) + \cos(4x-x))$

$= \frac{1}{4}(2\cos 8x\cos x + 2\cos 4x\cos x)$

$= \frac{1}{2}\cos x(\cos 8x + \cos 4x)$

$= \frac{1}{2}\cos x(\cos(6x+2x) + \cos(6x-2x))$

$= \frac{1}{2}\cos x(2\cos 6x\cos 2x)$

$= \cos 6x\cos 2x\cos x$

-RHS as required

(3) - correct solution

(2) - 1 error

(1) - ~~two~~ correct using product to sums

Done quite well by most students.

- A small number did not get to Q11 → ran out of time.

Suggested solutions	Marker's Feedback

11a(ii) Using (i) $0 \le x \le \pi$

$\cos 6x \cos 2x \cos x = 0$

$\cos 6x = 0, \quad 0 \le 6x \le 6\pi$

$6x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$

$x = \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12}$

← (1) – solves $\cos 6x = 0$

Also $\cos 2x = 0 \quad 0 \le 2x \le 2\pi$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

← (1) solves $\cos 2x = 0$

Also $\cos x = 0 \quad 0 \le x \le \pi$

$x = \frac{\pi}{2}$

← (1) – solves $\cos x = 0$

$\therefore x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{11\pi}{12}$

↑ $\frac{3\pi}{12}$ ↑ $\frac{9\pi}{12}$

A significant number of students did not find $0 \le x \le \pi$ ⇒ lost a mark.